DILATANCY BEHAVIOR IN CONSTANT STRAIN RATE CONSOLIDATION TEST

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ABSTRACT

Subjected to remolded young clay, this paper shows that a lot of time dependent behavior in the standard consolidation (SC) and constant strain rate consolidation (CSRC) tests is represented systematically by a simple assumption concerning the time dependency of dilatancy. In the SC test, at the first stage of each loading step little dilatancy takes place and dilatancy begins to occur several minutes after step loading. In CSRC test, some time period after the stress state has entered the normally consolidated region, dilatancy tends to occur rapidly with the increase in stress ratio. Since most of dilatancy has taken place at the earlier stage of consolidation, little dilatancy occurs at the latter stage of CSRC process. This tendency makes the specimen stiffer with the passage of time, and makes the vertical pressure and pore pressure increase substantially at the last stage of CSRC process. Consideration to such behavior may be effective to correctly interpret the result of CSRC test.

Keywords: clay, consolidation test, dilatancy, time effect.

INTRODUCTION

The constant strain rate consolidation (CSRC) test appears to be one of the most promising types of rapid consolidation test, because of its simple manipulation and reliability. A lot of research studies have enabled to get the void ratio-vertical effective stress relationship, coefficient of volume compressibility and coefficient of consolidation by using CSRC test results [1,2,3,4] These studies are divided broadly into two categories, according to whether the study assumes the linear elastic behavior of soils or not. The large strain effect has been considered also by some researchers [5,6]. However the time dependency in stress-strain response such as the secondary compression has not been sufficiently clarified yet in CSRC test. Although Leroueil et al [7] and Yin and Graham [8] practiced various types of oedometer tests including CSRC test and proposed a unique stress-strain rate relationship, the rheological for CSRC process itself are not explained in detail.

Previously Arai [9] has shown that both time effects observed in triaxial test and the secondary compression in standard consolidation test are realistically simulated by a numerical procedure based on a simple postulate concerning the time dependency of dilatancy. This paper applies the numerical procedure to CSRC test, and aims to clarify the time dependent behavior of soft clay in CSRC test. As the initial step of research study, the concern was restricted to only the stress-strain-time behavior observed in remolded young clay, and the infinitesimal strain theory is used.

DEFINITIONS

Fundamental Parameters

For the \( K_0 \) consolidation process, a stress state is described by mean effective stress \( p^n \) and stress difference \( q^n \):

\[
p^n = (\sigma_v^n + 2\sigma_h^n / 3),
\]

\[
q^n = \sigma_v^n - \sigma_h^n
\]

and the corresponding strains are

\[
v^n = \varepsilon_v^n,
\]

\[
\varepsilon^n = 2\varepsilon_v^n / 3
\]

\( \sigma_v^n, \sigma_h^n \) are the vertical and horizontal stresses respectively, \( \varepsilon_v^n \) the vertical strain, and \( n \) a discreti-
zed time step number. Hereafter regard normal stresses as effective stresses unless otherwise defined. The term $q''/p''$ is referred to 'stress ratio'.

**Time Dependency of Dilatancy**

When neglecting time dependency, volumetric strain due to dilatancy $v_d''$ is represented as [10, 11]

$$v_d'' = D q'' / p'' ,$$

where $D = (\lambda - \kappa) / (1 + e_0) M$ in which $D$ is the dilatancy coefficient, $\lambda = 0.434 C_c$ ($C_c$: compression index), $\kappa = 0.434 C_s$ ($C_s$: swelling index), $e_0$ initial void ratio, and $M$ slope of critical state line. When the stress state within clay specimen changes with the passage of time, it is assumed that volumetric strain due to dilatancy $v_d''$ takes place as

$$\Delta v_d'' = \left( D q'' / p'' - v_d''^{-1} \right) L''$$

(4)

in which

$$L'' = \ln \Delta T'/\ln T : \text{when the stress ratio increases}$$

$$= \Delta \ln t_i' / (\ln T - \ln t_i'^{-1}) : \text{for other cases}$$

where $\Delta t_i''$ the length of discretized time step, $t_i'$ the elapsed time after stopping the increase in the stress ratio, $T$ the time length required to make dilatancy take place completely, and $\Delta$ the increment of succeeding physical quantity. The physical meaning of Eqs. (4) and (5) is given in Arai [9]. Eq. (4) is not sufficient to duplicate the dilatancy behavior for a sudden increase in shear stress. When a shearing process originates in almost static state where the stresses and strain are approximately constant with the passage of time, for instance, the final stage of consolidation, it is assumed that little dilatancy is generated at the early stage of shear until the value of $v_d''$ calculated by Eq. (4) exceeds a certain prescribed value $v_d''^*$.

The value of $v_d''^*$ is easily determined using the effective stress path in conventional triaxial test [12]. The early stage of shear where little dilatancy takes place, is called 'quasi-elastic state'. Summarizing the above assumption, the time dependency of dilatancy is illustrated schematically in Fig. 1.

**Elastic and Plastic Strain**

The volumetric strain given by eq. (4) is assumed perfectly plastic. Incremental volumetric strain due to consolidation $\Delta v''_{ce}$ is supposed to be the sum of elastic component $\Delta v''_{ce}$ and plastic component $\Delta v''_{cp}$, which are assumed time independent.

$$\Delta v''_{ce} = \Delta v''_{ce} + \Delta v''_{cp}$$

(6)

$$\Delta v''_{ce} = \kappa (1 - e_0) \Delta p'' / p''$$

(7)

$$\Delta v''_{cp} = (\lambda - \kappa) (1 - e_0) \Delta p'' / p''$$

(8)

Although incremental shear strain, $\Delta \varepsilon''$, consist of elastic and plastic components, $\Delta \varepsilon''$ is specified by Eq. (10) for the $K_0$ consolidation process, without defining these components.

**TEST PROCEDURE**

The soil used for consolidation tests is the remolded Ota clay in Fukui Prefecture, Japan, which has the following index properties: specific gravity, 2.67; liquid limit, 53.2%; plastic limit, 25.4%; clay fraction, 60%; silt fraction, 36.7%; and fine sand fraction, 3.3%. The clay was mixed with water into slurry (moisture content = 72 %) and consolidated under the preconsolidation pressure of 98 kPa in a consolidometer. After being stored for more than two weeks, a test specimen 20 mm in height and 60 mm in diameter was carefully trimmed from the preconsolidated cake which was fully saturated. The standard consolidation test was carried out in accordance to the Japanese Industrial Standard A1217. The loading pressures were applied step-by-step as 9.8; 19.6; 39.2; 78.4; 156.8; 313.6; 627.2; 1254.4 kPa, each of which was maintained constant for 24 hours. CSRC test was carried out under the specified strain rates of 0.1, 0.05, and 0.01 %/min. CSRC test was performed by the equipment shown in Fig. 2.

During CSRC process the authors did not control the strain rate but deformation rate. The upper and lower surfaces of specimen are the pervious and impervious boundaries respectively. Pure water pressure is monitored at the bottom of specimen. After setting a test specimen into oedometer and after introducing the back pressure of 98 kPa, the specimen was consolidated by the vertical constant pressure of 9.8 kPa for 24 hours under the anisotropic $K_0$-condition.
NUMERICAL ANALYSIS

Discretization Technique

Beside treating $K_0$-consolidation here, the authors employ a finite element technique for two-dimensional consolidation analysis developed by Akai and Tamura [13], this technique seems to have attained a sufficiently reliable and popular stage. A consolidation test specimen subjected to analysis is subdivided into rectangular finite elements as shown in Fig. 3. The rectangular element is considered to be composed of four triangular elements, and the stresses, strains and pore water pressure are assumed to be constant throughout each rectangular element.

Nonlinear Stress-Strain Analysis

The authors concern are to solve the following two equations as a simultaneous equation, without using the plastic flow rule. One is an equation for specifying a volumetric strain due to consolidation and dilatancy (Eq. 9). The other is an equation which specifies a shear strain (Eq. 10). Assuming that $n^\prime \Delta v$ and $n q \Delta \varepsilon$ are given at a time step, the volumetric strain is calculated as the sum of the components due to consolidation and dilatancy, according to Otha [10].

$$\Delta v = \Delta v_{cc}^n; \text{ over-consolidated}$$

$$= \Delta v_{cc}^n + \Delta v_{cp}^n + \Delta v_{cd}^n; \text{ normally consolidated}$$

For the $K_0$-consolidation process, the shear strain is given as

$$\Delta \varepsilon = 2 \Delta \varepsilon / 3 = 2 \Delta v / 3$$

Using $\Delta v$ and $\Delta \varepsilon$, the hypothetical bulk modulus $K^n$ and shear rigidity $G^n$ are calculated as

$$\Delta p^m = K^n \Delta v^m, \Delta q^m = 3G^m \Delta \varepsilon^m$$

(11)

Based on $K^n$ and $G^n$, the hypothetical Young's modulus, $E^n$ and Poisson's ratio, $\nu^n$ are

$$E^n = 9K^n G^m / \left(3K^n + G^m\right),$$

$$\nu^n = \left(3K^n - 2G^m\right) / \left(6K^n + 2G^m\right)$$

(12)

Detailed reduction is given by Arai [7], These hypothetical elastic constants enable to relate all the stress components to strain components respectively without using the plastic flow rule.

Numerical Procedure

1) assume the trial values of hypothetical Young's modulus $E^m$ and Poisson's ratio $\nu^m$. 2) Using the assume $E^m$ and $\nu^m$ and known permeability $k^m$, perform the elastic consolidation analysis at a discretized time step. In the consolidation analysis, the displacements are specified for CSRC test and the loads are given for the standard consolidation test at the prescribed boundaries. 3) Employing the incremental stresses obtained by the consolidation analysis, the incremental volumetric and shear strains are calculated respectively from Eqs. (9) and (10). 4) Based on these strains, find the hypothetical elastic constants by using Eqs. (11) and (12). 5) replacing the trial values of $E^m$ and $\nu^m$ with those found at step 4), repeat the consolidation analysis until convergence of elastic constant is obtained. This iteration procedure is practiced for each discretized time step.
EXPERIMENTAL AND NUMERICAL RESULTS

Material Parameters

In the standard consolidation and CSRC tests, void ratio $e$, effective and vertical consolidation pressure $\sigma'_v$, coefficient of volume compressibility $m_v$, and coefficient of consolidation $C_v$ are calculated according to JIS (Japanese Industrial Standard) A1217 and JSF (Japanese Soil and Foundation) T412-1993 respectively. Fig. 4 shows $e - \log \sigma'_v$ relationship obtained by these two types of consolidation test, in which the consolidation yield stress is affected in some degree by the strain rate. Fig. 5 shows $m_v$ and $C_v$ obtained by the two types of consolidation test which appear to provide similar tendency. Fig. 6 illustrates the relationship between $e$ and permeability $k$ (cm/s) which is calculated from $C_v$ and $m_v$. As shown in Fig. 6, the two types of consolidation test seem to give a similar $e - \log k$ relationship. Except for exceedingly loose state, the relationship is approximately represented as:

$$e - 0.67 = 0.445 \left( \log_{10} k - \log_{10} \left( 8 \times 10^{-9} \right) \right) \quad (13)$$

Though it is not easy to find the correct permeability of cohesive soil due to many factors affecting the permeability [14, 15] we employ the above relationship, which provides the reasonable numerical results given later. The consolidation yield stress is regarded as $p_c = (98 + 2 K_0)/3 = 63$ kPa, where $K_0$ value is taken as 0.464, which is monitored by some $K_0$ consolidation tests using a triaxial equipment at the normally consolidated region [9]. Based on these results, the material parameters required for our numerical analysis are determined as $\lambda = 0.16$, $\kappa = 0.033$, $M=1.13$, $D=0.056$, $T=14$ days, $v'_w = 0.004$, $e_0 = 1.04$, and Eq. (13). The value of $\kappa$, $M$, and $v'_w$ are the same as those used in Arai [9] which are determined by triaxial compression tests performed separately.

Initial State

The authors numerical analysis treats the stress-strain response after placing a test specimen into oedometer. It is difficult to estimate the residual effective stresses or negative pore water pressure which remains within the specimen, since it is not easy to specify completely the stress history of the specimen until the specimen is placed into oedometer. The authors assume that 40% of mean effective stress in consolidometer remains isotropically within the specimen before starting the consolidation test, together with the same value of negative pore water pressure. It is also difficult to estimate the history of dilatancy before placing a test specimen into oedometer, because it is not easy to evaluate the dilatancy amount of slurry sample in consolidometer, and because the consolidated sample expands both vertically and laterally when taken out from the consolidometer. As the first approximation, on the analogy of the residual effective stresses described above assume that the dilatancy estimated by Eq (3). The authors has taken place almost entirely in consolidometer, and that 40% of dilatancy amount remains within the test specimen in oedometer. This rough approximation provides fairly good numerical results, and that the
numerical results are not so sensitive to the dilatancy amount before placing the specimen into oedometer.

**Standard Consolidation Test**

Fig. 7 compares the experimental and numerical results of time-settlement relationship in the standard consolidation test. The numerical result appears to agree fairly well with the experimental one, except for the loading pressure of 78.4 kPa, since we assume that no plastic component of volumetric strain takes place at the over-consolidated state. In this paper, only the stress ratio, volumetric strain and hypothetical elastic constant, $E_n$ and Poisson’s ratio, $\nu$ during consolidation are illustrated in Figs. 8 through 10 for comparing with the result of CSRC test.

In Figs. 8 through 10 and in Fig. 12 through 16 shown later, $B_i$ denotes the point at which the stress state moves to the normally consolidated region, and $C_i$ designates the point at which dilatancy begins to occur both in finite element $i$. As seen in Fig. 9, at the first stage of each loading step, the state of the specimen enters the quasi-elastic state illustrated in Fig. 1, where no dilatancy takes place. A little time after each step loading, the volumetric strain due to dilatancy begins to occur.

Although the stress ratio becomes almost constant at the latter stage of each loading step as shown in Fig. 8, the ratio continues to increase very slightly with the passage of time. Referring to Figs. 7 through 9, the secondary compression observed at the latter stage of standard consolidation test, is represented by the dilatancy occurrence according to nearly constant stress ratio, which take place proportionally with the logarithm of elapsed time. Figs. 9 and 10 indicates the following tendency of hypothetical Young’s modulus $E_n$. After the stress state moves to normally consolidated region, $E_n$ decreases suddenly due to the occurrence of $\nu$. Subsequently $E_n$ increases gradually with the progress of consolidation. The occurrence of dilatancy reduces $E_n$ substantially, because the volumetric due to dilatancy increases whereas the effective vertical stress little increases at latter stage of each stage of each loading step. Whether dilatancy occurs or not, $E_n$ approaches 0 at the final stage of each loading step, because the effective stresses reach the constant values in spite of the slight increase in volumetric...
strain. Hypothetical Poisson’s ratio, $\nu^n$ is held constant throughout the consolidation process shown in Fig. 10.

![Figure 10. Hypothetical elastic constants (SC test)](image)

**Constant Strain Rate Consolidation (CSRC) test**

After setting a test specimen into oedometer, the specimen is consolidated by the total vertical pressure of 9.8 kPa for 24 hours under $K_0$ condition. Following this preconsolidation process, CSRC test is carried out under a certain strain rate prescribed. Thus our numerical analysis is performed by two separate stages, i.e. the preconsolidation process for which the preconsolidation pressure of 9.8 kPa is specified, and CSRC process for which a displacement is prescribed at the upper surface of specimen at each discretized time step. The initial state for preconsolidation process is the same as the initial state for the standard consolidation test described previously.

The final state for preconsolidation process corresponds to the initial state for CSRC process. Note that at the final state of the preconsolidation process, vertical stress $\sigma'_v$ (9.8 kPa) is less than lateral stress $\sigma'_h$ (18.03 kPa), due to the isotropic initial stresses of 24 kPa for the preconsolidation process. Fig. 11 makes comparison between the calculated and monitored quantities for CSRC process, according to the strain rate. As seen in Fig. 11, the prescribed and monitored displacements at the upper surface of specimen are slightly different from each other due to the limitation of displacement control system. In our numerical analysis, the actually monitored displacement is given at each discretized time step. Thus the fluctuation from prescribed displacement causes the fluctuation of physical quantities calculated by the proposed procedure (for instance, see Figs. 12 through 15). As shown in Fig. 11, both the calculated loading pressure, displacement and pore water pressure appear to agree fairly well with the monitored results. Figs. 12 through 16 illustrated some of the detailed numerical results upon which the calculated results shown in Fig. 11 are based. Though it is not possible to compare directly these detailed numerical results with monitored ones, these numerical results enable to interpret conscientiously the soft clay behavior during CSRC process, which is described as follows.

In Figs. 12 through 16, vertical stress $\sigma'_v$ is less than lateral stress $\sigma'_h$ until point $\Lambda_i$, where suffix $i$ denotes finite element number. From the initial state to point $\Lambda_i$, $q^n$ increases as shown in Fig. 16 because $\sigma'_v$ approaches gradually $\sigma'_h$, resulting in increase $q^n/p^n$ as shown in Fig. 13. At this state, Poisson’s ratio is kept constant and $E^n$ increases slightly with the progress of consolidation. $\Delta q^n/\Delta p^n$ is held constant throughout CSRC process as illustrated in Fig. 16. This makes $q^n$ or $\sigma'_v$ increase, and makes $E^n$ increase as shown in Fig. 16 because of the small total volumetric strain which consists of only $cev^n$.

Note that the constant value of $\Delta q^n/\Delta p^n$ provides a constant Poisson’s ratio of 0.317 after passing point $\Lambda_i$ as shown in Fig. 15. Although $\Delta q^n/\Delta p^n$ is constant for CSRC process, stress ratio $q^n/p^n$ varies as illustrated in Fig. 16, and approaches to the following value as shown in Fig. 13.

$$ (q^n/p^n) = \left( \frac{\sigma'_v - \sigma'_h}{\sigma'_h + 2\sigma'_h} \right) = 0.834 \text{ (14)} $$

In Figs. 12 through 16, at point $B_i$, the stress of element $i$ moves to the normally consolidated region. Immediately after passing point $B_i$, the state enters the quasi-elastic state at which $vcv^n$ and $vcp^n$ take place. This transition causes the reduction of $E^n$ as in Fig. 15 because of the considerable increase in volumetric strain, which restrain the increase $q^n$ or stress ratio $q^n/p^n$ as shown in Fig. 13.
Figure 11. Comparison between calculated and monitored results (CSRC Test)

Figure 12. Pore water pressure

Figure 13. Stress ratio

Figure 14. Components of volumetric strain
The duration of quasi-elastic state is short in this case, because the stress ratio tends to increase rapidly for CSRC process. Thus the quasi-elastic state may not give a substantial effect to the global behavior of CSRC specimen. Immediately after the state of specimen has removed from the quasi-elastic state (point $C_i$ in Figs. 12 through 16), volumetric strain due to dilatancy $v_d^n$ reduces $E^n$ as shown in Fig. 15. Subsequently $E^n$ tends to increase gradually, because the occurrence of dilatancy becomes smaller with the passage of time as shown Fig. 14. Note that we assume that the total amount of dilatancy is limited by Eq. (3) as illustrated in Fig. 14. Since most of dilatancy has taken place at the earlier stage of consolidation, little dilatancy occurs at the latter stage of CSRC process. This means that larger effective stresses are required to produce the constant volumetric strain at the latter stage where the volumetric strain consist of almost only $v_{ce}^n$ and $v_{cp}^n$. This tendency makes the specimen stiffer with time (see Fig. 15) and makes loading pressure and pore water pressure increase substantially as illustrated in Fig. 11. Being similar with the standard consolidation test, when the increasing rate of stress ratio is less than $10^{-3}$/min, Eq. (5b) is employed instead of Eq. (5a). This treatment is applied only to the initial and last stages where dilatancy takes place for strain rate $= 0.01 \%$/min, since the loading pressure for stress ratio continues to steadily increase in most stages of CSRC process. In Figs. 12 through 16, the difference of physical quantities in each finite element is remarkable for strain rate $= 0.1 \%$/min due to less uniform distribution of pore water pressure within a specimen, while the difference seems almost negligible for $0.01 \%$/min (see Fig. 17). When the strain rate is too fast, and when the difference of physical quantities within a specimen is larger as seen in Figs. 13 through 16, the overall material parameters obtained from CSRC test which are supposed constant within a test specimen, may tend to fluctuate largely. For instance, Fig. 18 shows $m_v$ and $c_v$ in each finite element calculated by using hypothetical elastic constants $E^n$ and $v_v^n$. The general agreement between Figs. 18 and Fig. 5, indicates the appropriateness of the proposed procedure. The difference of $E^n$ in each finite element as shown in Fig. 15, appears to provide the unstable variation of overall $m_v$ and $c_v$ as shown in Fig. 5 which are assumed constant within a specimen, especially for the over-consolidated ratio. The secondary compression coefficient is defined for nearly constant stress ratio such as the latter stage of each loading step in standard consolidation test as illustrated in Fig. 8. It may be difficult to find directly the secondary compression coefficient by using CSRC process, and because most of dilatancy has taken place when the stress ratio becomes approximately constant at the last stage of CSRC test as shown in Figs. 13 and 14.
CONCLUDING REMARKS

Subjected to remolded young clay, this paper has shown that a lot of time dependent behavior in the standard consolidation and CSRC tests is represented by a simple assumption concerning the time dependence of dilatancy. This paper assumes the time dependency of dilatancy as shown in Fig. 1 and Eqs. (4) and (5). The behavior in the standard consolidation test is as follows. At first stage of each loading step the state of specimen is at the quasi-elastic state where no dilatancy take place. A little time after step loading, dilatancy begins to occur. Because the stress ratio becomes approximately constant at the latter stage of each loading step, dilatancy occurs proportionally with the logarithm of elapsed time, which is observed as the secondary compression.

The dilatancy behavior in CSRC test is as follows. When the stress state enters the normally consolidated region, the state moves to the quasi-elastic state. Since the duration of quasi-elastic state is short in CSRC test, the state does not give a dominant effect to the global behavior of test specimen. Some time period after the stress state has entered the normally consolidated region, dilatancy tends to occur rapidly with the increase in stress ratio. Since most of dilatancy has taken place at the earlier stage of consolidation, little dilatancy occurs at the latter stage of CSRC process. This tendency makes the specimen stiffer with the passage of time, and makes the vertical pressure and pore pressure increase substantially at the last stage of CSRC process. Considerations to such behavior may be effective to correctly interpret the result of CSRC test. The secondary compression coefficient is defined for nearly constant stress ratio such as the latter stage of each loading step in standard consolidation test. It may be difficult to find directly the secondary compression coefficient by using CSRC test, because the stress ratio continues to considerably change throughout CSRC process, and because most of dilatancy has taken place when the stress ratio becomes approximately constant at the last stage of CSRC test.

The assumption of time dependency of dilatancy enables to simulate many time effects in standard consolidation and CSRC tests. However, by using only this assumption, it is not possible to simulate the time dependent behavior such as the change of consolidation yield stress according to the loading duration in the standard consolidation test or the strain rate in CSRC test. Although such behavior may be deeply related to time dependency of dilatancy, it may require another assumption concerning the time dependency of volumetric strain due to isotropic consolidation, which is an important subject to be investigated in the future study.

REFERENCES


